# Year 9 Sequences and Functions – Revision Notes

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Year 9 · Maths · Sequences and Functions · Key ideas and worked examples

**For grown-ups and learners:** read each part and try the worked example. Then work through the sheets: Foundation, Developing, Challenge, then Mixed Practice to revise everything. Show your working: it earns marks in tests and helps you spot mistakes.

## 1. Term-to-term rules and quadratic sequences

A **term-to-term rule** can use any operations, including powers: start at 1 with 'square, then add 1' to get 1, 2, 5, 26 and so on. In a **quadratic sequence** the first differences change but the **second differences** are constant: 2, 6, 12, 20, 30 has differences 4, 6, 8, 10 and second difference 2. Continue the differences to find later terms.

**Example:** The first term of a sequence is 1. The term-to-term rule is 'multiply by 3, then add 2'. Write the first five terms.
→ Apply the rule to each term in turn: 1 → 5 → 17
→ → 53 → 161
**Answer: 1, 5, 17, 53, 161**

## 2. Rules for the nth term

If the difference is a (any number, including negatives and fractions), the nth term is **an ± b**: 7, 4, 1, −2, ... goes down by 3, so the rule is −3n + 10. Some sequences match special rules ($n^2$, $n^2 \pm a$, $n^3$ and n/a). The rule **$n^2$** gives 1, 4, 9, 16, ... and **$n^2 + 3$** gives 4, 7, 12, 19, .... Similarly, **$n^3$** gives 1, 8, 27, 64, ... and **n/5** gives 1/5, 2/5, 3/5, ....

**Example:** Find the nth term of the sequence 11, 5, −1, −7, ...
→ Difference −6, so the rule starts −6n
→ n = 1 gives −6; first term 11
→ −6n + 17
**Answer: −6n + 17**

## 3. Functions and inverses

A **function** gives exactly one output for each input. $y = 2x^2 - 3$ gives 15 when x = 3 (and also when x = −3: different inputs may share an output). To find an input, use **inverse operations in reverse order**: for input → cube → + 5 → output 69, subtract 5 (64) and take the cube root (4). Undoing a square gives two possible inputs, one positive and one negative.

**Example:** The function machine is input → cube → + 4 → output. The output is 129. Find the input.
→ Undo + 4: 129 − 4 = 125
→ Undo the cube: $\sqrt[3]{125} = 5$
**Answer: 5**

## 4. Is it a term?

To test whether a number is in a sequence, set the nth term equal to it and solve. Is 150 a term of 4n − 2? 4n − 2 = 150 gives n = 38, a whole number, so **yes, it is the 38th term**. Is 100 a term of $n^2 + 5$? $n^2 = 95$, and 95 is not a square number, so **no**. The position n must always be a positive whole number.

**Example:** Is 75 a term of the sequence with nth term $n^2 - 8$? If it is, which term is it?
→ Solve $n^2 - 8 = 75$
→ n is not a whole number: no
**Answer: No: solving $n^2 - 8 = 75$ does not give a whole number n.**

**Think like a mathematician:** test a few cases (**specialising**), look for the rule that always works (**generalising**), and explain why you are sure (**convincing**). When you check someone's working, say what is wrong and how to **improve** it.

**Common mistakes:** writing n + 4 for a sequence that goes up in 4s. Adding the same difference to a quadratic sequence. Squaring the 2 in $2n^2$. Doing inverse operations in the wrong order. Forgetting the negative input when undoing a square.