Year 9 Indices and Surds – Revision Notes

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Year 9 · Maths · Irrational Numbers, Indices and Surds · Key ideas and worked examples

For grown-ups and learners: read each part and try the worked example. Then work through the sheets: Foundation, Developing, Challenge, then Mixed Practice to revise everything. Show your working: it earns marks in tests and helps you spot mistakes.

1. Negative and zero indices

Look at the pattern $2^3 = 8$, $2^2 = 4$, $2^1 = 2$, $2^0 = 1$: each time the index goes down by 1, the value is divided by 2. Carry on and $2^{-1} = 1/2$, $2^{-2} = 1/4$. So a **negative index** means the **reciprocal**: $5^{-2} = 1/5^2 = \mathbf{1/25}$, and $(2/3)^{-1} = 3/2$. A negative index never makes the number negative. Any number except 0 to the power **0** is **1**.

Example: Work out $4^3 \times 4^{-4}$. Give your answer as a fraction.
→ $4^3 \times 4^{-4} = 4^{-1}$
→ $= 1/4$
Answer: 1/4

2. Index laws with any integer index

For powers of the **same base**, multiply by **adding** the indices and divide by **subtracting** them. The laws still work with negative indices: $3^5 \times 3^{-8} = 3^{5+(-8)} = \mathbf{3^{-3}}$, and $7^2 \div 7^{-4} = 7^{2-(-4)} = \mathbf{7^6}$. Take care when you subtract a negative index: it adds.

Example: Write $11^5 \times 11^{-4} \div 11^3$ as a single power of 11.
→ Add, then subtract the indices: $5 + (-4) - 3 = -2$
→ $= 11^{-2}$
Answer: $\mathbf{11^{-2}}$

3. Rational and irrational numbers

A **rational** number can be written as a fraction of two integers: 0.75 = 3/4, −5 = −5/1, 0.333... = 1/3 and √49 = 7 are all rational. An **irrational** number cannot be written as a fraction: its decimal goes on for ever with no repeating pattern. √2, √50, $\sqrt[3]{10}$ and π are irrational. The square root of a whole number is rational only when the number is a square number. A **surd** is an irrational root such as √7.

4. Estimating surds

Use the square numbers either side. $6^2 = 36$ and $7^2 = 49$, so **$\sqrt{40}$ is between 6 and 7**. For 1 decimal place, square decimals: $6.3^2 = 39.69$ and $6.4^2 = 40.96$, so √40 is between 6.3 and 6.4. Then check the halfway value: $6.35^2 = 40.3225$ is more than 40, so **$\sqrt{40} \approx 6.3$**. Cube roots work the same way with cube numbers: $4^3 = 64$ and $5^3 = 125$, so $\sqrt[3]{100}$ is between 4 and 5.

Example: Find $\sqrt{35}$ correct to 1 decimal place, without a calculator. Use squares of decimals to check.
→ $5.9^2 = 34.81$ and $6^2 = 36$, so $\sqrt{35}$ is between 5.9 and 6
→ $5.95^2 = 35.4025$, which is more than 35, so $\sqrt{35} \approx 5.9$
Answer: 5.9

Think like a mathematician: test a few cases (**specialising**), look for the rule that always works (**generalising**), and explain why you are sure (**convincing**). When you check someone's working, say what is wrong and how to **improve** it.

Common mistakes: writing $2^{-3} = -8$ instead of 1/8. Multiplying the indices instead of adding them. Thinking $\sqrt{(9 + 16)} = 3 + 4$. Thinking every square root is irrational ($\sqrt{64} = 8$ is rational).